Key

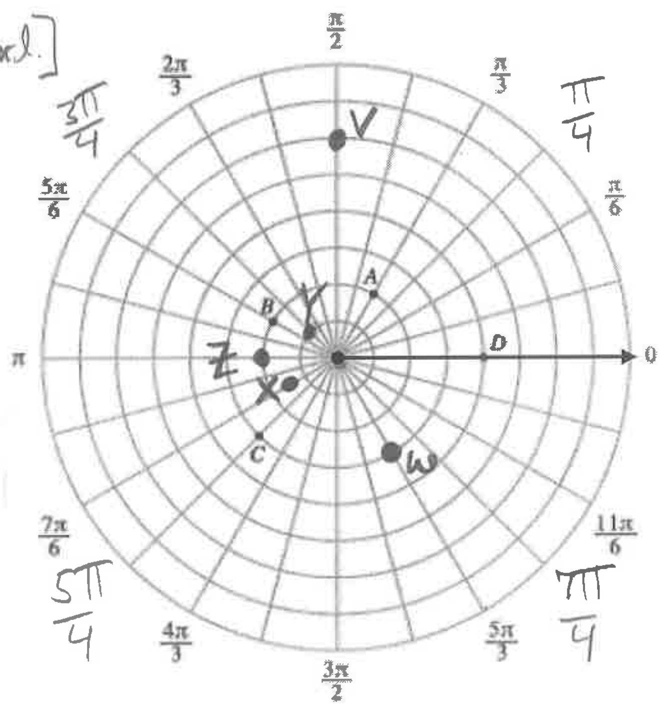
Math 4 Name ____________________

Polar Graphing Activity Date ________

For years, you have been plotting function equations on a Cartesian coordinate grid. In this activity, you will learn about the polar coordinate system and graphing polar equations. The polar coordinate system consists of a fixed point called the *pole* (similar to the origin in a Cartesian coordinate system) and a fixed ray called the *polar axis*. A polar coordinate graph is usually made on a grid that shows concentric circles that have the pole at the center and rays that begin at the pole and are associated with different amounts of turn. Each point in the polar coordinate system is identified by its distance from the pole, represented by r, and an angle of turn from the polar axis, represented by θ. The polar coordinates of a point are given in the form $[r, \theta]$. The coordinate r is called the *radial coordinate* and the coordinate θ is called the *angular coordinate*. When θ is positive, the angle is measured in the counterclockwise direction and when θ is negative, the angle is measured in the clockwise direction.

I. On the polar grid below, there are four points that have already been graphed.

[radial coordinate, angular coord.]



1. The point A can be represented by the coordinates $\left[2, \frac{\pi}{3}\right]$ or the coordinates $\left[2, \frac{7\pi}{3}\right]$. Explain why these two coordinate pairs represent the same point and give another coordinate pair for this point.

   $\frac{7\pi}{3}$ is like making one full rotation + an extra $\frac{\pi}{3}$. $\left[2, -\frac{5\pi}{3}\right]$

2. Point B can be represented by the coordinates $\left[2, \frac{-7\pi}{6}\right]$. Represent point B using a positive value for θ.

   $\left[2, \frac{5\pi}{6}\right]$ or $\left[2, \frac{17\pi}{6}\right]$

3. On a polar graph, a point's distance from the pole will always be a positive number. Because of this, plotting points with a negative r value is not completely straightforward. The point C can be represented by the coordinates $\left[3, \frac{5\pi}{4}\right]$ as well as $\left[-3, \frac{\pi}{4}\right]$. Give **two** different sets of polar coordinates for point D. Make sure one set has a negative radial value.

   $[4, 0]$ + $[-4, \pi]$

OVER →

4. Plot the following points on the polar coordinate grid on the front of this packet. Then provide **one** other coordinate pair that represents the same point.

Answers vary!

$V\left[6, \frac{\pi}{2}\right] = \left[-6, \frac{3\pi}{2}\right]$

$W\left[3, \frac{-\pi}{3}\right] = \left[3, \frac{5\pi}{3}\right]$

$X\left[1.5, \frac{7\pi}{6}\right] = \left[-1.5, \frac{\pi}{6}\right]$

$Y\left[-1, \frac{3\pi}{4}\right] = \left[-1, \frac{11\pi}{4}\right]$

$Z\left[2, 5\pi\right] = \left[-2, 0\right]$

5. Explain why every point on a polar coordinate system can be identified with an infinite number of coordinates.

You can rotate around the circle infinitely!

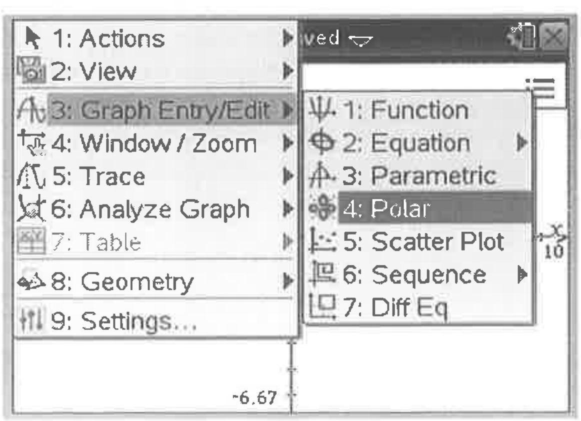
II. In the next part of this activity, you will explore graphs of polar equations. A polar equation is a function rule in the form $r = f(\theta)$, where θ can be measured in radians or degrees.

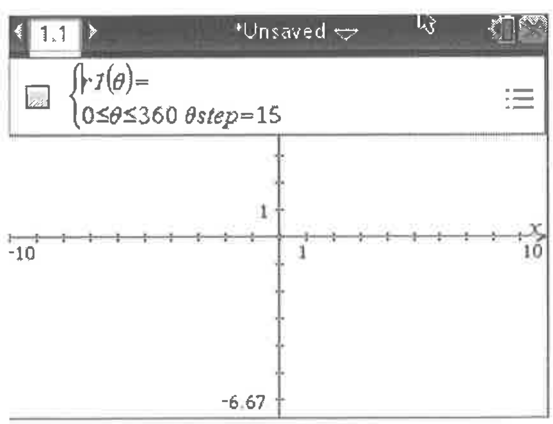
What is the *independent* variable? θ

What is the *dependent* variable? r

Switch your calculator into degree mode. (or scratchpad) On graph page, Menu-Settings

To graph in the polar coordinate system using your calculator, use these settings:





Use your calculator to explore the following:

1. Consider equations of the form: $r = a\sin\theta$ / $r = a\cos\theta$ Experiment with different values for a.

a. What type of figure is created by these equations? Circles

b. How do the figures differ when different trig functions are used (sin vs. cos)?

Sine graphs are centered on the y-axis.
Cosine " " " " " x-axis.

c. What is significant about the a-value?

It is the radius of the circle.

2. Consider equations of the form: $r = a \pm b\sin\theta$ / $r = a \pm b\cos\theta$. ***Limaçons***

Graph together: $r = 2 + 5\sin\theta$, $r = 1 + 3\cos\theta$ Graph together: $r = 4 + 3\sin\theta$, $r = 3 + 2\cos\theta$ Graph together: $r = 4 + 4\sin\theta$, $r = 2 - 2\cos\theta$

➢ How do the figures differ when different trig functions are used (sin vs. cos)?

Same as #1b.

3. Consider equations of the form: $r = a\sin(n\theta)$ / $r = a\cos(n\theta)$ ***Rose Curves***

[If n is even, # of petals = 2n
If n is odd, # of petals = n]
[a determines length of petals.]

Graph these functions one at a time: $r = 2\sin(3\theta)$ $r = 4\sin(2\theta)$ $r = 2\cos(3\theta)$ $r = 4\cos(2\theta)$

a. How do the figures differ when different trig functions are used (sin vs. cos)?

The figures rotate slightly when changing from sine to cosine.

b. What determines the length of a petal?

The a-value.

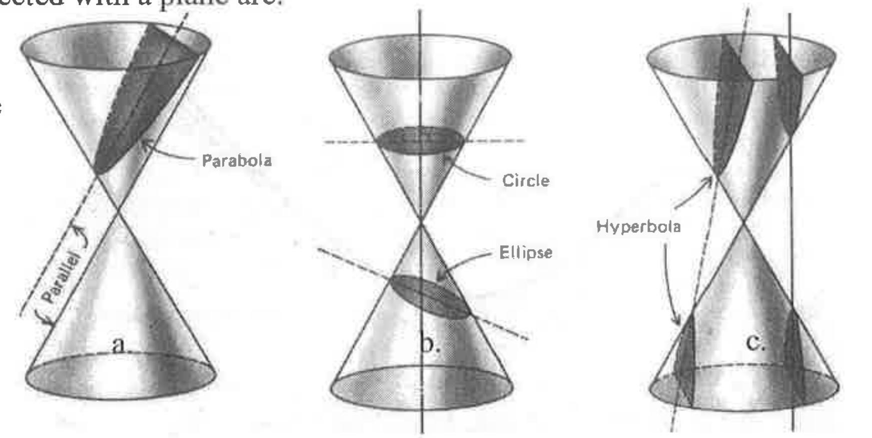
c. What determines the number of petals?

The n-value. See notes above.

OVER →

4. In mathematics, a **conic section** (or just **conic**) is a curve obtained by intersecting a cone (more precisely, a right circular conical surface) with a plane. The three conic sections that are created when a double cone is intersected with a plane are:

a. Parabola
b. Circle and ellipse
c. Hyperbola



Source: http://project1.caryacademy.org/Earthquakes/2005/HA218/images/conic.jpg

Each of the above curves can be represented in rectangular form as follows:

Circle: $x^2 + y^2 = a^2$ Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Parabola: $y^2 = 4ax$. Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

How might you graph these in your calculator? Think polar! Try this:

Reset the window of your calculator to that of the beginning of part II. Graph each of the following, one at a time.

$$r = \frac{10}{1 + 3\cos\theta} \qquad r = \frac{1}{3 + 2\cos\theta} \qquad r = \frac{1}{2 - 2\cos\theta}$$

(Ignore any lines; they're asymptotes)

➤ What is the name of the shape for each figure produced?

1st: Hyperbola 2nd: Ellipse 3rd: Parabola

***type in like: 10 sin(θ) (cos(θ))^2

5. Graph each equation below.

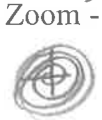
a. $r = 10\sin(\theta)\cos^2(\theta)$ This is called a ***Bifolium***. Why?

The graph has two loops.

Menu-4-A

b. $r = \frac{2\theta}{\pi}$ To see this graph better: Change θmax to 3600; then Zoom - Fit.

FYI - This called a ***Spiral of Archimedes.***



c. $r = 3\csc(\theta) + 5$ Use the same settings as those from part b.

This is called the ***Conchoid of Nichodemes.*** What happens if this is in terms of the secant?

The figure rotates 90° Goes from horizontal to vertical.

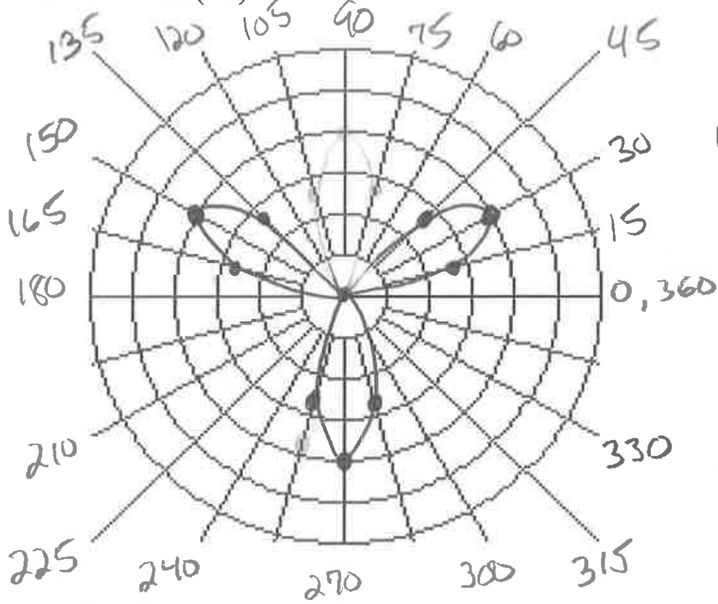
Polar Graphing Practice

Change θ Max & Step to same as Part II.
Then Menu-4-5 (Zoom-Standard)

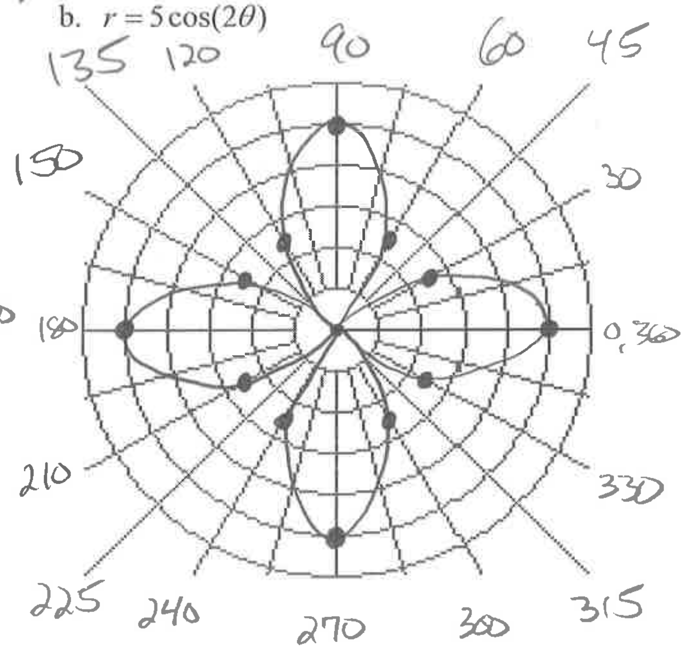
1. Sketch accurate graphs of the following. ***Use a table to help you plot your points.***

Table: Ctrl-T

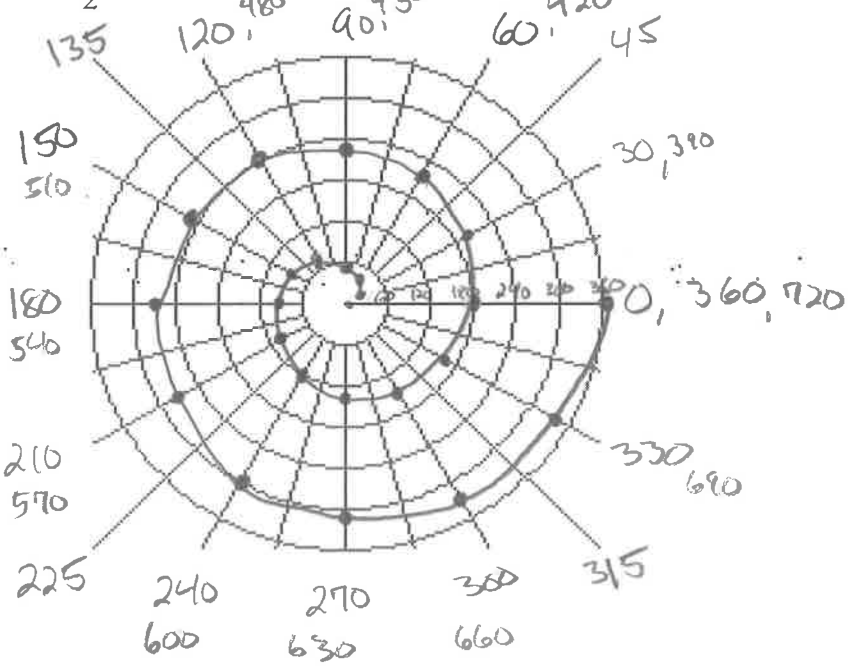
a. $r = 4\sin(3\theta)$



b. $r = 5\cos(2\theta)$



c. $r = \dfrac{\theta}{2} + 3$ (Graph 2 revolutions.)



X & y scale should be about 60ish.

Answer the following:

2. Ralph was confused about how the polar equations above can be considered functions when their graphs do not pass the vertical line test.

   ➢ Use the definition of function to explain why each polar equation ***does*** represent a function.

Function: every X corresponds to exactly one y-value.

For polar: every θ corresponds to exactly one r-value.
Since $\frac{\pi}{3}$ & $\frac{7\pi}{3}$ are the same location, you can have circles & spirals that are still functions.